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Physical System Parameters

Drone Parametrization

v 0.4

What this document covers

This document covers both the linear and non-linear mapping from the Generalized Forces to the Motor Commands, that are used in the Autonomous Vehicles Research Studio documentation and software.

System Parameters

Some QDrone parameters that are used in the derivations in this document have been listed in Table 1 below.

Table 1: QDrone mechanical parameters		
Dimensions		
L_{roll}	Roll motor-to-motor distance	0.2136 m
L_{pitch}	Pitch motor-to-motor distance	0.1758 m
$L \times W \times H$	Net frame dimensions along x, y & z directions	0.363 m x 0.403 m x 0.139 m
h_{cg}	Vertical C.O.G location (w.r.t the ground when drone sits flat)	0.051 m
Mass and Moment of Inertia		
M_b	Battery mass	0.267 kg
M_d	Drone mass	0.854 kg
M	Total mass	1.121 kg
J_{xx}	Roll Moment of Inertia	1.0×10^{-2} kgm^2
J_{yy}	Pitch Moment of Inertia	8.2×10^{-3} kgm^2
J_{zz}	Yaw Moment of Inertia	1.48×10^{-2} kgm^2
Electrical Parameters used in linear mapping		
A_{hover}	Hover motor current	5.82 A
k_t	Motor torque constant	4.50×10^{-3} Nm/A
K_v	Motor speed constant (specification)	2100 RPM/V

Linear Force to Motor Command Mapping

Given the weight of the QDrone and the nominal full voltage of a 3S LiPo battery (12.6V), the selected propulsion system on the QDrone is able to maintain hover at around 53.8% command (u_{hover}) on a fully charged fresh battery. If linear extrapolation is used, the maximum commanded thrust force for each motor (thrust at 100% command) can be estimated by taking the QDrone takeoff weight, divided by 4 times the hover percentage command:

$$K_f = F_{max} = \frac{M \times g}{4 \times u_{\text{hover}}}$$

$$K_f = \frac{(0.854 + 0.267) \times 9.81}{4 \times 0.538} = 5.11N \tag{1}$$

Where u_{hover} was obtained from experimental data collected in a manual flight. Roll and Pitch moment arms are estimated as half the motor-to-motor distances. They are obtained by measuring the distance between the center of the motor shafts along the forward/backward and sideway direction of the QDrone frame:

$$L_{roll} = 0.2136m$$
$$L_{pitch} = 0.1758m \tag{2}$$

The motor torque constant k_t is obtained from the motor manufacturer data sheet. The hover motor current A_{hover} is obtained from experimental data collected with dynamometer. The normalized yaw torque constant K_t can be estimated as follows

$$K_t = k_t \frac{A_{hover}}{u_{hover}}$$
$$K_t = 4.5 \times 10^{-3} \frac{Nm}{A} \frac{5.8239A}{0.538} \tag{3}$$
$$K_t = 0.0492Nm$$

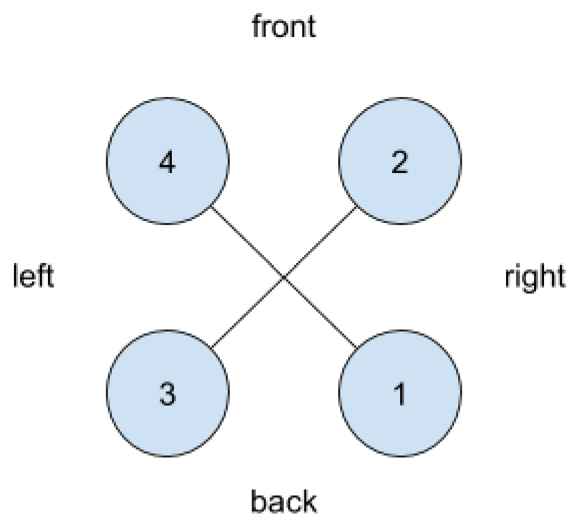

Figure 1: Motor ordering in the QDrone

The mapping matrix, from motor percentage command u_i to system force/torques,

$$\begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} = \begin{bmatrix} K_f & K_f & K_f & K_f \\ -K_f\frac{L_{roll}}{2} & -K_f\frac{L_{roll}}{2} & K_f\frac{L_{roll}}{2} & K_f\frac{L_{roll}}{2} \\ K_f\frac{L_{pitch}}{2} & -K_f\frac{L_{pitch}}{2} & K_f\frac{L_{pitch}}{2} & -K_f\frac{L_{pitch}}{2} \\ K_t & -K_t & -K_t & K_t \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{4}$$

Inverting the mapping matrix to obtain the inverse relationship:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4K_f} & -\frac{1}{2K_f L_{roll}} & \frac{1}{2K_f L_{pitch}} & \frac{1}{4K_t} \\ \frac{1}{4K_f} & -\frac{1}{2K_f L_{roll}} & -\frac{1}{2K_f L_{pitch}} & -\frac{1}{4K_t} \\ \frac{1}{4K_f} & \frac{1}{2K_f L_{roll}} & \frac{1}{2K_f L_{pitch}} & -\frac{1}{4K_t} \\ \frac{1}{4K_f} & \frac{1}{2K_f L_{roll}} & -\frac{1}{2K_f L_{pitch}} & \frac{1}{4K_t} \end{bmatrix} \begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} \tag{5}$$

Thus, a final controller command as a thrust force (N) and 3 rotation torques (Nm) can be mapped to a set of motor percentage commands (% PWM pulse from 0 to 1) using the matrix in (5) above.

The maximum system force/torque (mapping to 100% command) will be [20.44 N, 1.09 Nm, 0.90 Nm, 0.10 Nm] for Throttle thrust, Roll torque, Pitch torque and Yaw torque (TRPY) with a 3S LiPo battery, 6045 props and 2206 Cobra motors (2100 Kv)..

This is linearly extrapolated about the hover force of 11.0N (53.8% of maximum thrust force of 20.44N).

Figure 2 below shows the linear and non-linear models compared. The non-linear mapping is presented in the next section.

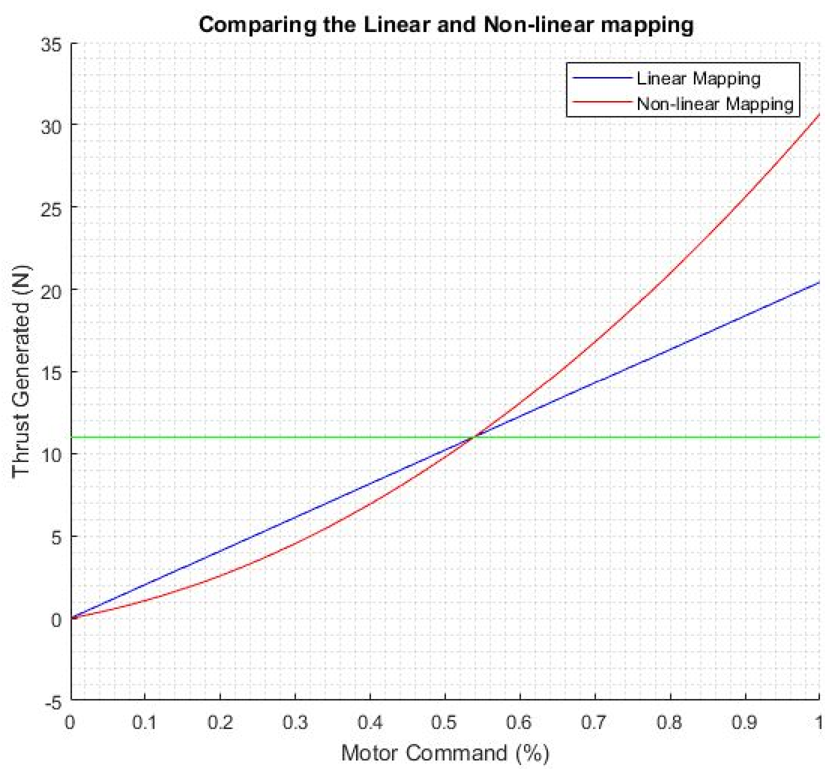

Figure 2: Linear vs. Non-linear mapping models

Non-Linear Force to Motor Command Mapping

Experimentally, the relationship between the applied motor command (%) and the corresponding command voltage applied to the motors by the ESC is,

$$u = \frac{V}{V_d} \tag{6}$$

Where V_d is the battery voltage. The angular velocity of the propeller (6045 durable polycarbonate) is linearly related to the commanded voltage as,

$$V = \frac{1}{K_{v,eff}}(\omega - \omega_c) \tag{7}$$

Here, ω_c is an angular velocity offset in RPM and $K_{v,eff}$ is the effective motor speed constant in RPM/V. The parameters ω_c and $K_{v,eff}$ are obtained by fitting a linear polynomial (Figure 3 linear fit) to angular velocity (RPM) and voltage command (V) data collected from a test on a dynamometer (Figure 3 raw data).

The thrust F_m produced by the rotating propeller has a squared relationship with the angular velocity of the propeller, and can be experimentally estimated as,

$$F_m = C_t(\omega + \omega_f)^2 + F_b \tag{8}$$

Where C_t is the motor force constant in N/RPM^2 of the motor/propeller combination, ω_f is another angular velocity offset and F_b is the force offset in N. The parameters C_t and F_b are obtained by fitting a quadratic polynomial (Figure 4 quadratic fit) to thrust (N) and angular velocity (RPM) data collected from hover flights of the QDrone with a varying payload (Figure 4 raw data).

Thus, the commanded force for each motor can be mapped to the commanded voltage. The required voltage corresponds to a motor command that compensates for the current battery voltage level.

Note: The angular velocity offset ω_c is obtained experimentally by fitting a linear polynomial to the voltage command vs. angular velocity curve. This results in a non-zero angular velocity at a zero voltage command. This non-zero angular velocity will map to a non-zero thrust, which, is not practical for use in our control model, where a zero voltage command should be mapped to a zero thrust generated. Thus, another angular velocity offset ω_f is introduced. Here, ω_f was calculated using the equation,

$$\omega_f = \sqrt{\frac{-F_b}{C_t}} - \omega_c$$

which is obtained by solving equations 7 and 8 for ω_f with $V = 0$ and $F_m = 0$. This is also illustrated in Figures 3 and 4 below.

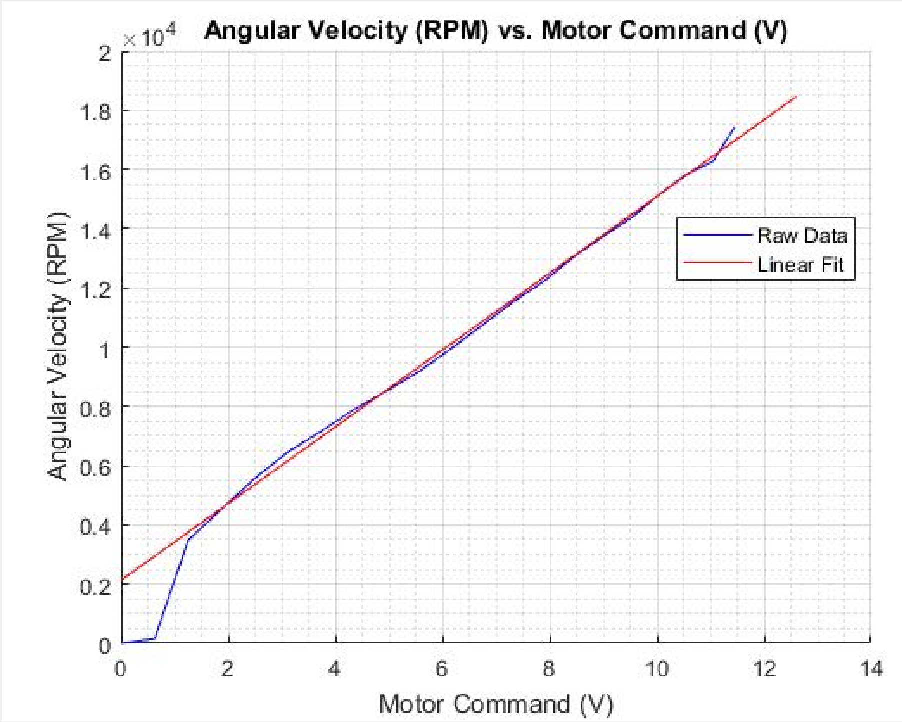

Figure 3: Motor Command (V) vs. Angular Velocity (RPM) - angular velocity of 2132.6 RPM at a 0 voltage command

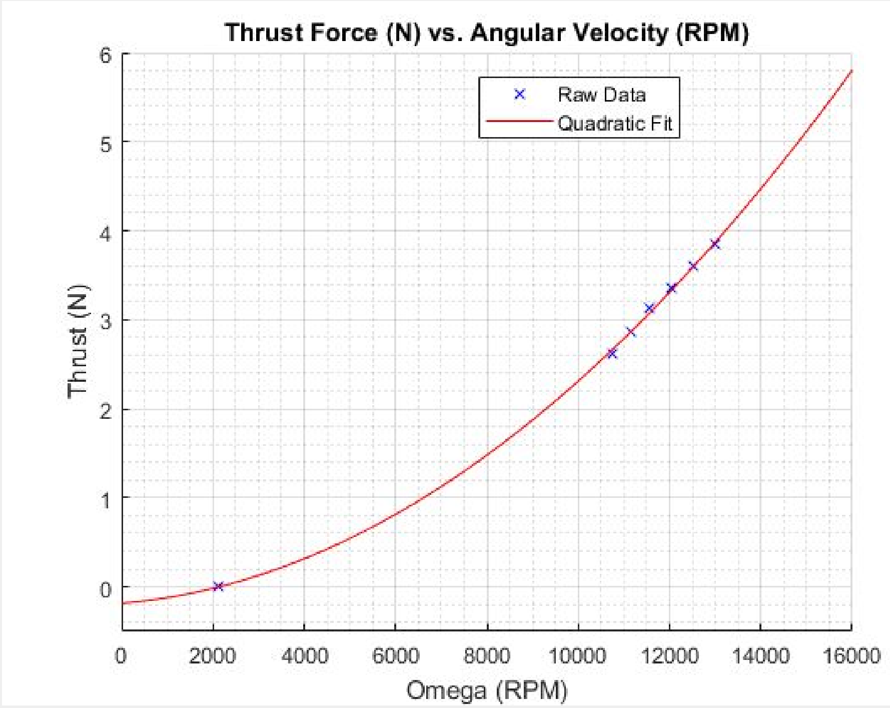

Figure 4: Angular Velocity (RPM) vs. Thrust Generated per Motor (N) - 0 thrust generated at An angular velocity of 2132.6 RPM

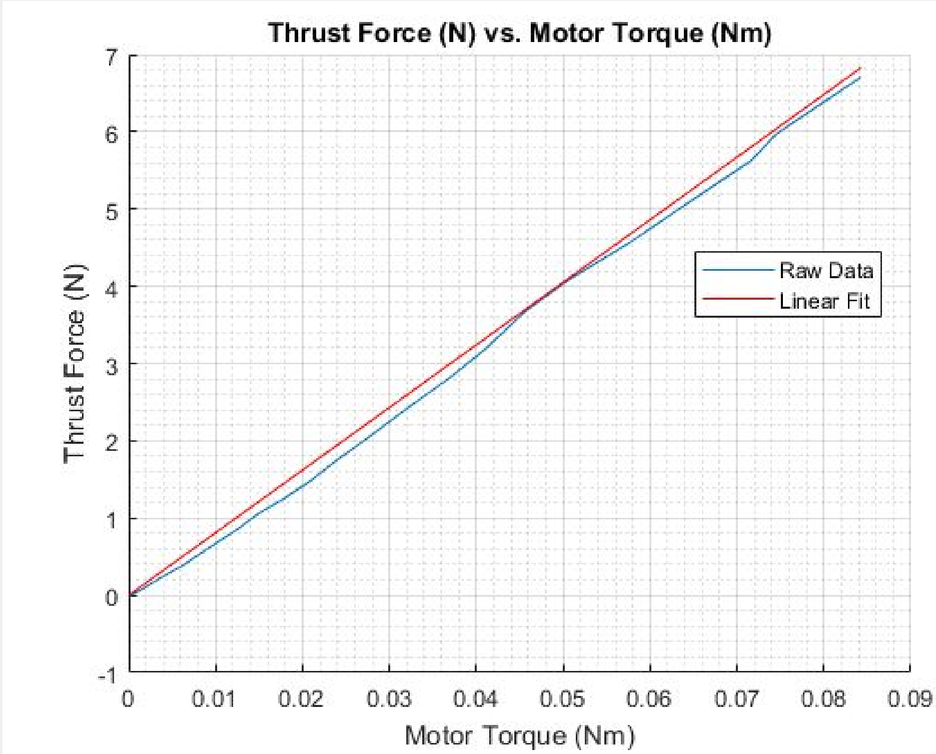

Figure 5: Motor Torque (Nm) vs. Thrust Generated per Motor (N)

Finally, the motor torque τ_m is linearly related to the motor force F_m by

$$F_m = k_\tau \tau_m \tag{9}$$

Where k_τ is the motor thrust-torque constant. This is obtained by fitting a linear polynomial (Figure 5 linear fit) to the motor torque vs. thrust generated data (Figure 5 raw data).

The parameters $K_{v,eff}$, ω_c, ω_f, C_t, F_b and k_τ obtained have been summarized in Table 2 below.

Table 2: QDrone non-linear model parameter estimates		
Dimensions		
$K_{v,eff}$	Effective motor speed constant	1295.4 RPM/V
ω_c	Voltage to Angular velocity offset	2132.6 RPM
ω_f	Angular velocity to force offset	1004.5 RPM
C_t	Motor force constant	2.0784 x 10^{-8} N/RPM^2
F_b	Motor force offset	-0.2046 N
k_τ	Motor thrust-torque constant	81.0363 N/Nm

Thus, given the generalized force vector, one can find the corresponding motor forces as,

$$\vec{F}_m = \begin{bmatrix} F_{m,1} \\ F_{m,2} \\ F_{m,3} \\ F_{m,4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & \frac{k_\tau}{4} \\ \frac{1}{4} & -\frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & -\frac{k_\tau}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & \frac{1}{2L_{pitch}} & -\frac{k_\tau}{4} \\ \frac{1}{4} & \frac{1}{2L_{roll}} & -\frac{1}{2L_{pitch}} & \frac{k_\tau}{4} \end{bmatrix} \begin{bmatrix} F \\ \tau_{roll} \\ \tau_{pitch} \\ \tau_{yaw} \end{bmatrix} \tag{10}$$

From here, the angular velocity is obtained by,

$$\vec{\omega} = \sqrt{\frac{1}{C_t}\left(\vec{F}_m - \begin{bmatrix} F_b & F_b & F_b & F_b \end{bmatrix}^T\right)} - \begin{bmatrix} \omega_f & \omega_f & \omega_f & \omega_f \end{bmatrix}^T \tag{11}$$

where $\vec{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^T$. From here, the required motor voltage vector is given by,

$$\vec{V} = \frac{1}{K_{v,eff}}\left(\vec{\omega} - \begin{bmatrix} \omega_c & \omega_c & \omega_c & \omega_c \end{bmatrix}^T\right) \tag{12}$$

Where $\vec{V} = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}^T$ is a vector of motor voltages. Lastly, the required motor command is then,

$$\vec{u} = \frac{1}{V_d}\vec{V} \tag{13}$$

Thus, a final controller command as a thrust force (N) and 3 rotation torques (Nm) can be converted to a set of motor commands (% PWM pulse from 0 to 1) using the equations in (10) and (11) above.

The maximum system force/torque will be [30.67 N, 1.6373 Nm, 1.3476 Nm, 0.1892 Nm] for Throttle thrust, Roll torque, Pitch torque and Yaw torque (TRPY) with a 3S LiPo battery, 6045 props and 2206 Cobra motors (2100 Kv). Note that this mapping results in a trim of 53.8% as well.